Solutions to JEE Advanced - 1 | Paper-1 | JEE 2024

PHYSICS

SECTION-1

1.(ABC) Capacitance with slab inside,

$$C_K = \left(\frac{d/2}{\varepsilon_0 A} + \frac{d/2}{K\varepsilon_0 A}\right)^{-1} \quad \Rightarrow \quad C_K = \left(\frac{2K}{K+1}\right)\frac{\varepsilon_0 A}{d} = \left(\frac{2K}{K+1}\right)C$$

Charge on plates when charged to potential V,

$$Q = \left(\frac{2K}{K+1}\right)CV_0$$

Charge remains the same after slab is removed

Now, during removal of the slab, let:

Work done by external force $= W_{ext}$

Change in capacitor energy $= \Delta U$

Conserving energy,

$$W_{ext} = \Delta U \quad \Rightarrow \quad W_{ext} = \frac{1}{2}\frac{Q^2}{C} - \frac{1}{2}\frac{Q^2}{C_K}$$

$$= \frac{1}{2}\left(\frac{2K}{K+1}\right)^2 C^2 V_0^2 \left[\frac{1}{C} - \frac{K+1}{2KC}\right] \qquad = \frac{1}{2}\frac{2K(K-1)}{(K+1)^2}CV_0^2$$

Therefore, $$W_{ext} = \frac{K(K-1)}{(K+1)^2}CV_0^2$$

And, the p.d. across the plates after the slab is removed is $V = \dfrac{Q}{C} = \left(\dfrac{2K}{K+1}\right)V_0$

2.(BD) If the potential difference across a cylindrical conductor is V, the current through it, $i = \dfrac{V}{\left(\dfrac{\rho L}{A}\right)}$

The drift speed through the conductor, $v_d = \dfrac{i}{nAe} = \dfrac{V}{ne\rho L}$

And, the rate of heat of heat dissipation in the conductor, $H = Vi = \dfrac{V^2 A}{\rho L}$

Now, since the conductors A and B are in parallel, the potential difference across them is equal. Also, since they are made of the same material, the values of resistivity, ρ and the number of charge carriers per unit volume, n are also equal. Therefore,

$$\frac{v_A}{v_B} = \frac{L_B}{L_A} = 2$$

Also, $$\frac{H_A}{H_B} = \left(\frac{A_A}{A_B}\right)\left(\frac{L_B}{L_A}\right) = (2)^2(2) = 8$$

3.(BC) The given charge distribution is a superposition of:

(a) a uniformly charged sphere of radius R centred at origin with charge density $+\rho$

(b) a unifomly charged sphere of radius r centred at $\left(0, 0, \frac{R}{2}\right)$ with charge density $-\rho$

Electric field:

at (0, 0, R) $\quad \vec{E}_A = \left(\frac{\rho R}{3\varepsilon_0} + \frac{-\rho r^3}{3\varepsilon_0\left(\frac{R}{2}\right)^2}\right)(\hat{k}) \quad = \frac{\rho R}{3\varepsilon_0}\left[1 - \frac{4r^3}{R^3}\right]\hat{k}$

at (0, 0, $-R$) $\quad \vec{E}_B = \left(\frac{\rho R}{3\varepsilon_0} + \frac{(-\rho)r^3}{3\varepsilon_0\left(\frac{3R}{2}\right)^2}\right)(\hat{k})\,;\ \vec{E}_B = \frac{\rho R}{3\varepsilon_0}\left(1 - \frac{4r^3}{9R^3}\right)(-\hat{k})$

Potential:

at (0, 0, R) $\quad V_A = \frac{\rho R^2}{3\varepsilon_0} + \frac{(-\rho)r^3}{3\varepsilon_0\left(\frac{R}{2}\right)} = \frac{\rho R^2}{3\varepsilon_0}\left(1 - \frac{2r^3}{R^3}\right)$

at (0, 0, $-R$) $\quad V_B = \frac{\rho R^2}{3\varepsilon_0} + \frac{(-\rho)r^3}{3\varepsilon_0\left(\frac{3R}{2}\right)} = \frac{\rho R^2}{3\varepsilon_0}\left(1 - \frac{2r^3}{3R^3}\right)$

SECTION-2

4.(B) $Ig(G + R_1) = 3 \quad \ldots$(i)

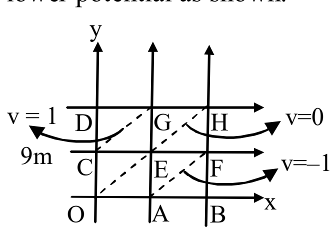
$Ig(G + R_1 + R_2) = 15 \quad \ldots$(ii)

$Ig(G + R_1 + R_2 + R_3) = 150 \quad \ldots$(iii)

Solving, we get $R_1 = 2.98\,k\Omega$

$R_2 = 12\,k\Omega; \quad R_3 = 135\,k\Omega$

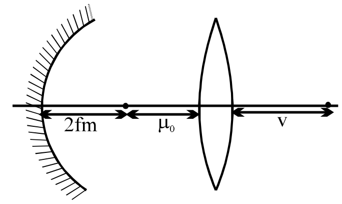
5.(B) OEH is an equipotential surface, the uniform E.F. must be perpendicular to it pointing from higher to lower potential as shown.



Hence $\hat{E} = \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right); \quad E = \frac{(v_E - v_B)}{EB} = \frac{0 - (-2)}{\sqrt{2}} = \sqrt{2}$

$\therefore \quad \vec{E} = E\,.\,\hat{E} = \sqrt{2}\,\frac{(\hat{i} - \hat{j})}{\sqrt{2}} = \hat{i} - \hat{j}$

6.(B) The object must be at centre of curvature of mirror $V = 1.5\mu_0$



$$\frac{1}{1.5\mu_0} - \frac{1}{\mu_0} = \frac{1}{10} \qquad \therefore \qquad \mu_0 = \frac{50}{3}$$

Again $\quad 2f_m + \frac{50}{3} = 40$

$$\therefore \qquad f_m = \frac{1}{2}\left(40 - \frac{5}{3}\right) = \frac{70}{6}$$

7.(B) Longest wavelength $n = 2$ to $n = 1$

$$\frac{mv^2}{r} = \frac{k.Ze(2e)}{r^2}; \qquad mvr = \frac{nh}{2\pi}$$

Kinetic energy, potential energy and total energy becomes four times

$$\therefore \qquad \Delta E \text{ becomes four times} \qquad \therefore \qquad \Delta E = 4Z^2Rch\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\Delta E = 3RchZ^2 \quad \Rightarrow \quad \frac{hc}{\lambda} = 3RchZ^2 \quad \Rightarrow \quad \lambda = \frac{1}{3Z^2R}$$

SECTION-3

8.(B) Due to original system $B_0 = 0$ and $B_P = \frac{\mu_0 I}{2\pi R}$

Removed part carries a total current of $I/4$

P. $\quad B_{net} = 0 - 0$

Q. $\quad B_{net} = = \frac{\mu_0 I}{2\pi R} - B'$ where $B'(2\pi R) = \mu_0\left(\frac{I}{4}\right)$

$$\Rightarrow B_{net} = \frac{3\mu_0 I}{8\pi R}$$

R. $\quad B_{net} = \left|0 - \frac{\mu_0\left(\frac{I}{4}\right)}{2\pi(R/2)}\right| = +\frac{\mu_0 I}{4\pi R}$

S. $\quad B_{net} = \frac{\mu_0 I}{2\pi R} - B'$ where $B'\left(2\pi \times \frac{3R}{2}\right) = \mu_0\left(\frac{I}{4}\right)$

$$\Rightarrow B_{net} = \frac{5}{12}\frac{\mu_0 I}{\pi R}$$

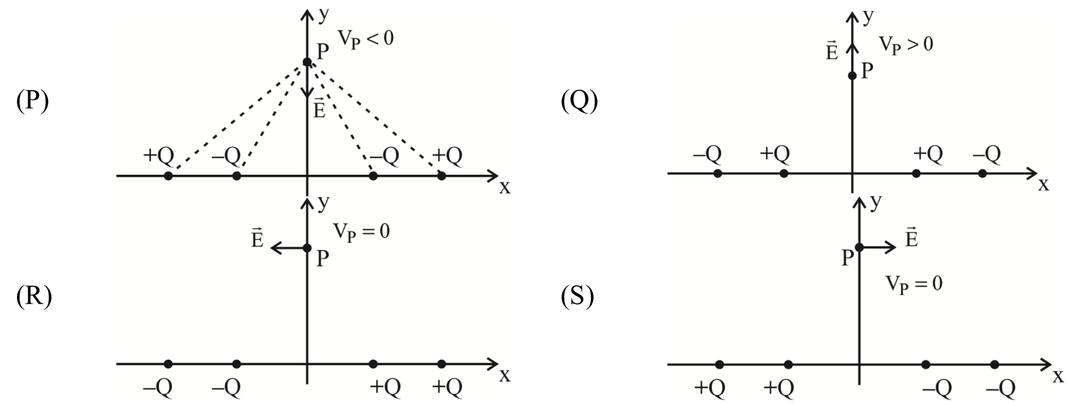
9.(C) **(A)** Potential energy stored in the capacitor will dissipate due to the wire.

(B) Motional emf generated and charges will collect at the end of the rods according to Lorentz force. Some amount of heat is produced due to motion of charges.

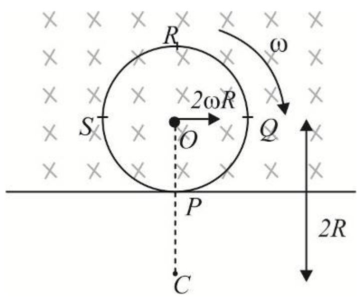
(C) Due to constant electric field all the electrons will move against the field and positive charge will appear at the other end, till the time net electric field across the wire becomes 0 net potential difference will appear across the ends. Some amount of heat is produced due to motion of charges.

(D) Due to battery a constant current will start flowing through the wire power is generated by the resistant of wire. Potential difference across the battery will be equal to the potential difference across the wire.

10.(B)



11.(B)



C = Instant centre of rotation

P. $V_{PR} = \frac{B\omega}{2}\left[(CR)^2 - (CP)^2\right] = 2BRV$

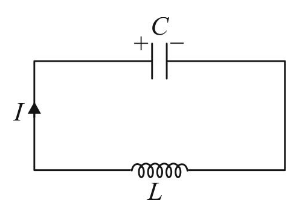
Q. $V_{PS} = \frac{B\omega}{2}\left[(CS)^2 - (CP)^2\right] = BRV$

R. $V_{PO} = \frac{B\omega}{2}\left[(CO)^2 - (CP)^2\right] = \frac{3}{4}BRV$

S. $V_{RO} = \frac{B\omega}{2}\left[(CR)^2 - (CO)^2\right] = \frac{5}{4}BRV$

SECTION-4

1.(3) Let the instantaneous charge on the capacitor be Q and let the instantaneous current be I



Then, $I = +\frac{dQ}{dt}$

From KVL: $-L\frac{dI}{dt} - \frac{Q}{C} = 0$

Combining the equations, we get

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$$

Therefore, $Q(t) = Q_{\max}\sin(\omega t + \phi)$

Here, $\omega = \frac{1}{\sqrt{LC}}$

So, $I(t) = \frac{dQ}{dt} = \omega Q_{\max}\cos(\omega t + \phi)$

We are given that at $t = 0,\ q = CV_0$ and $I = 0$, where $V_0 = 60$ volt

Therefore, $Q_{\max} = CV_0$ and $\phi = \dfrac{\pi}{2}$

So, $Q(t) = CV_0 \cos(\omega t)$ and $I(t) = -\omega CV_0 \sin(\omega t)$

Therefore, $I_{\max} = \omega CV_0$

$$= \frac{1}{\sqrt{LC}} CV_0 = \sqrt{\frac{C}{L}} V_0 = \sqrt{\frac{10^{-5}}{4\times 10^{-3}}}(60) = 3A$$

Alternative solution

Mathematically, an LC circuit is identical to an ideal spring-block oscillator. The charge on the capacitor is analogous to the displacement of the block (and hence the elongation of the spring), and the current in the circuit is analogous to the velocity of the block.

Therefore, connecting a charged capacitor to an inductor, and closing the switch with the current initially zero is exactly like releasing a block from rest with the spring elongated by a certain distance.

So, just like for a spring-block oscillator, the speed of the block (or its kinetic energy) is maximum when the elongation in the spring is zero, the current in an LC circuit (or the energy stored in the inductor, $\frac{1}{2}LI^2$) is maximum when the capacitor is completely uncharged.

Therefore, by energy conservation

$$\frac{1}{2}LI_{\max}^2 = \frac{1}{2}\frac{Q_{\max}^2}{C} \quad \Rightarrow \quad I_{\max} = \frac{Q_{\max}}{\sqrt{LC}} = \frac{CV_0}{\sqrt{LC}} = \sqrt{\frac{C}{L}}V_0 = 3\ A$$

2.(30) Intensity of EM wave, $I = \dfrac{1}{2\mu_0} B_0^2 c$ where B_0 is the amplitude of the magnetic field

So $$I = \frac{1}{2\left(1.25\times 10^{-6}\right)}\left(5\times 10^{-7}\right)^2\left(3\times 10^8\right) = 30\ W/m^2$$

3.(40) Since the current leads the voltage, the capacitor's reactance $\dfrac{1}{\omega C}$ is greater than the inductor's reactance ωL

Hence, $$\frac{\frac{1}{\omega C} - \omega L}{R} = \tan\phi = 1$$

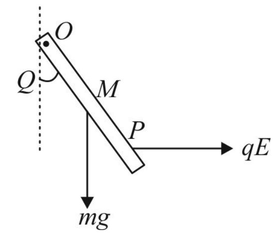
Solving, we get $L = 40$ mH

4.(28) Since field is uniform, the force can be found by replacing the portion of the loop inside the field by a straight wire between its end points.

$$\text{Force} = BI \times \left(\text{length } PQ\right)$$
$$= BI \times (PM + MN + NQ)$$
$$= BI \times \left(\frac{4R}{5} + 4R + \frac{4R}{5}\right)$$
$$= \frac{28}{5} BIR$$

5.(2) Since the charge is uniformly distribued, the point of application of the electric force (for the purpose of calculating torque) is the mid-point of the charged portion, P.

$$OP = \frac{1}{2}\left[\frac{2\ell}{3} + \ell\right] = \frac{5}{6}\ell$$



Balancing torque about O,

$$mg\left(\frac{\ell}{2}\right)\sin\theta = qE\left(\frac{5\ell}{6}\right)\cos\theta$$

$$\Rightarrow \qquad \tan\theta = \frac{5}{3}\frac{qE}{mg} = \frac{5}{3} \times \frac{\left(10^{-3}\right)(1200)}{(1)(10)} = 0.2$$

6.(26) Resistance of each bulb, $\qquad R = \frac{40^2}{4} = 400\ \Omega$

Let the number of connected bulbs be n

Then, the current through the battery,

$$i = \frac{60}{8 + \frac{400}{n}} = \frac{15n}{2n + 100}$$

Hence, the potential difference across each bulb,

$$V = i\left(\frac{400}{n}\right) = \frac{3000}{(n + 50)}$$

Therefore, the power consumed by each bulb,

$$P = \frac{V^2}{400} = \frac{22500}{(n + 50)^2}$$

Now, $\qquad P < 4 \quad \Rightarrow \quad (n + 50)^2 > \frac{22500}{4} \qquad \Rightarrow \qquad n > 25$

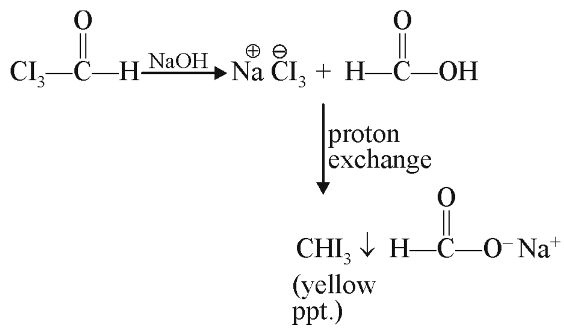
CHEMISTRY

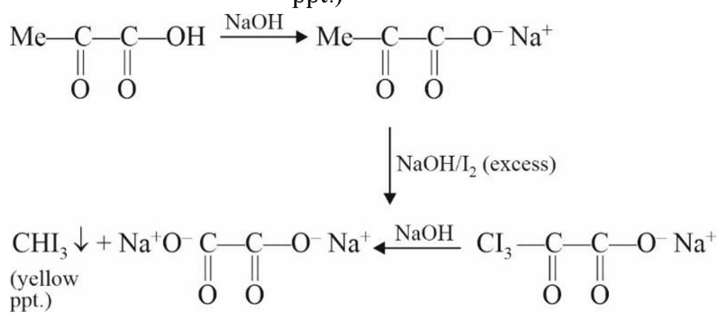
SECTION-1

1.(CD) In (C), if $CH_3-\overset{\displaystyle CH_3}{\underset{\displaystyle CH_3}{\overset{|}{\underset{|}{C}}}}-Cl$ is taken at starting material, alkene will be major product. In (D) both the groups are vinylic and vinyl halides do not undergo substitution.

2.(ABC)

3.(ABCD)

(B) 

(C) 

SECTION-2

4.(B) CH_3MgI on reaction with active hydrogen containing compounds gives methane gas

$ROH + CH_3MgI \rightarrow CH_4 + Mg(OR)I$

General molecular formula for saturated monohydric alcohol is $C_nH_{2n+2}O$ or $C_nH_{2n+1}OH$

Moles of methane formed = Moles of active hydrogen

= Moles of saturated monohydric alcohol reacted

Or, $\frac{44.48}{22400} = \frac{0.12}{14n+18}$

On solving for n

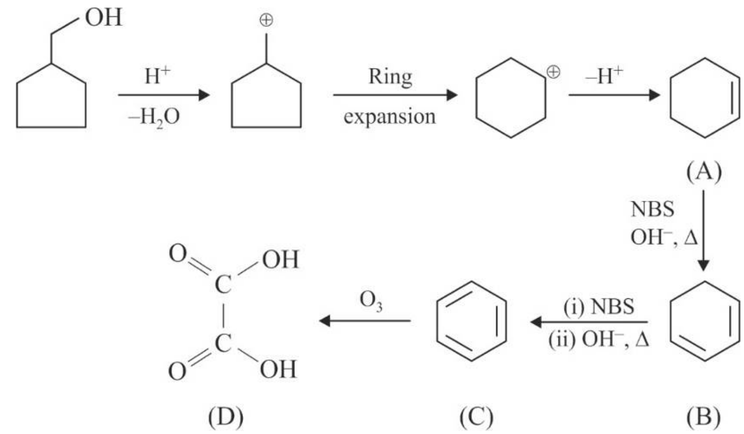
$n = 3$

Molecular formula of alcohol is C_3H_7OH

As it gives yellow precipitate of iodoform with I_2 and alkali, it should be $CH_3CH(OH)CH_3$

5.(C)

6.(D)



7.(B) $\therefore$ Moles of HIO_4 used = 2.

$x = 2$

SECTION-3

8.(B) For 1st order reaction

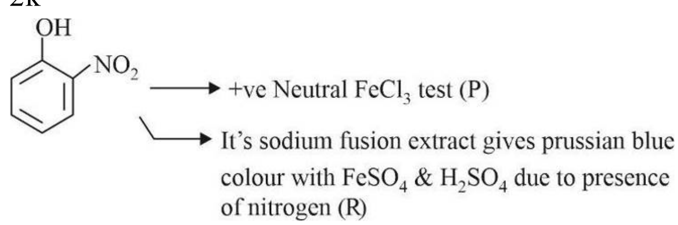
$$[A] = [A_0]e^{-kt}$$

$$t_{1/2} = \frac{\ln 2}{k}$$

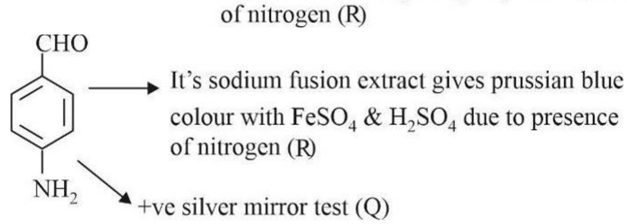
For zero order reaction

$$t_{1/2} = \frac{[A_0]}{2k}\ ;\ r = k$$

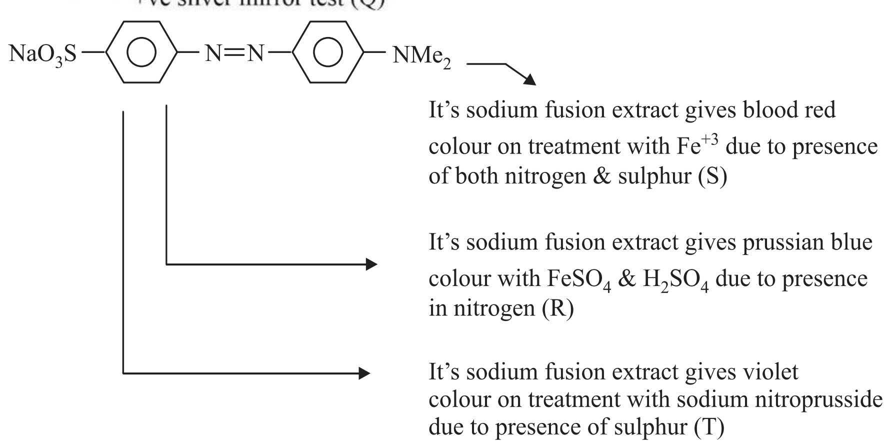
9.(D) I.



II.



III.



IV. $X : HCOOH \longrightarrow$ gives +ve silver mirror test (Q)

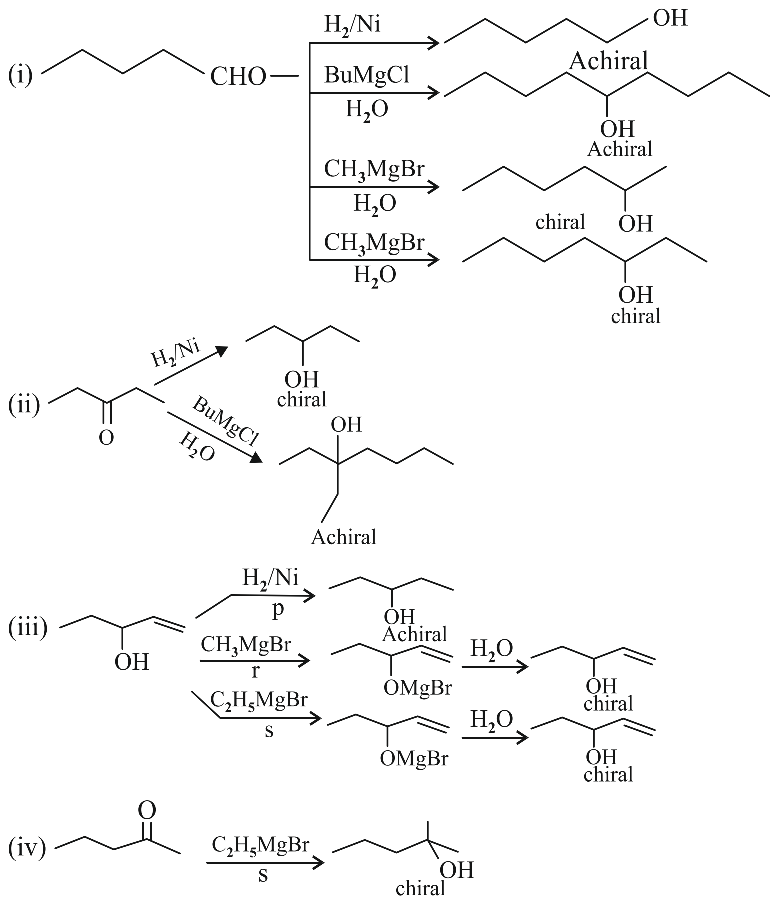
10.(A) (A) CO is strong field ligand and thus compels for pairing of electrons. Hence hybridization is sp^3 and complex is diamagnetic. Ligand is two electron donor.

(B) When NO molecule co-ordinates with metal atom to form metallic nitrosyls, the single electron present in π^* antibonding molecular orbital is transferred to metal atom M so that NO molecules is converted in to NO^+. Since NO^+ is isoelectronic with CO molecule, this ion coordinates with M^- ion as a two electron donor in metal nitrosyls in the same way as CO coordinate to M atom in metal carbonyls. Note that NO molecule is a three electron donor. Now the empty π^* antibonding molecular orbital can overlap with the filled d-orbital of metal to form $M^- \rightarrow NO^+ \pi$ bond.

(C) $\left[Ni(PF_3)_4\right]$ has bonding like that of $\left[Ni(CO)_4\right]$

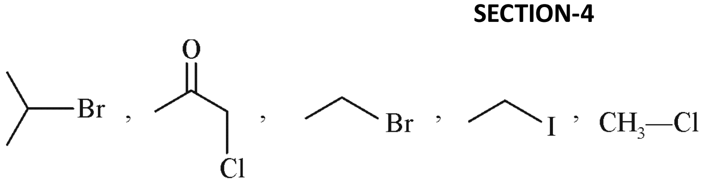
(D) Complex, $\left[PtCl_3(C_2H_4)\right]^-$ has $5d^8$ electron configuration. It is diamagnetic and square planar. There is π back donation between metal ion and ethylene.

11.(A)



SECTION-4

1.(5)



are more reactive.

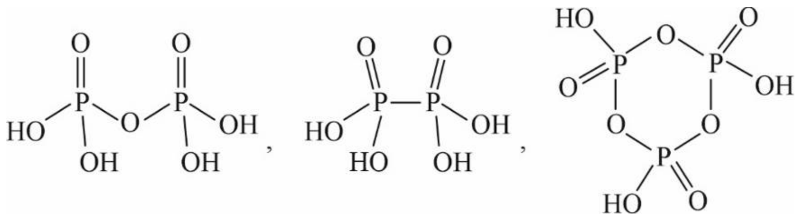
2.(3) Reaction (I), (II), (V) result in the formation of propanoic acid.

3.(9) $P_4 + 3NaOH + 3H_2O \xrightarrow{\text{Disproportionation reaction}} PH_3 + 3NaH_2PO_2$

4.(3) Reaction (b), (c), (d) will give salicylic acid as major product.

5.(5) (iii), (iv), (v), (vi), (x) will undergo solvolysis reaction faster than benzyl chloride as they form more stable carbocation than benzyl carbocation.

6.(11)



MATHEMATICS

SECTION – 1

1.(ABD) $\because$ $gof(x)$ is continuous $\therefore$ $g(x)$ and $f(x)$ are also continuous

Then $f(0) = \lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} f(0-h)$ $\Rightarrow$ $1 = \lim\limits_{h\to 0}(0-h+a) = a$

and $g(0) = \lim\limits_{x\to 0^-} g(x)$ $\Rightarrow$ $1+b=1$ $\Rightarrow$ $b=0$

$$\therefore \quad f(x)=\begin{cases} x+1 & , \ x<0 \\ |x-1| & , \ x\ge 0 \end{cases} \quad \text{and} \quad g(x)=\begin{cases} x+1 & , \ x<0 \\ (x-1)^2 & , \ x\ge 0 \end{cases}$$

$$g\left(f(x)\right)=\begin{cases} x+2 & x<-1 \\ x^2 & -1\le x<1 \\ (x-2)^2 & x\ge 1 \end{cases}$$

2.(BC) To limit exist and equal to 1

Coefficient of x^4 in denominator = 0

Coefficient of x^3 in numerator = 0

$$\frac{\text{coefficient of } x^2 \text{ in numerator}}{\text{coefficient of } x^2 \text{ in denominator}} = 1$$

Gives,

$$5a - b + 4c = 0$$
$$2a + b - 3c = 0$$
$$a - 5b + c + 2 = 0$$

Solving $a = \frac{-2}{109}; b = \frac{46}{109}; c = \frac{14}{109}$

3.(AD) $y = e^{\frac{3\sin^{-1}x}{4}+\frac{\pi}{8}}$

Maximum when $\sin^{-1}x = \frac{\pi}{2}$

Minimum when $\sin^{-1}x = -\frac{\pi}{2}$

$$y = e^{\frac{-3\pi}{8}+\frac{\pi}{8}} = e^{-\frac{\pi}{4}}$$

SECTION-2

4.(C) Given that $f(x) = |1-x|$

$$\Rightarrow \quad f(|x|) = \begin{cases} x-1, & x>1 \\ 1-x, & 0<x\le 1 \\ 1+x, & -1\le x\le 0 \\ -x-1, & x<-1 \end{cases}$$

Clearly the domain of $sin^{-1}(f|x|)$ is $[-2, 2]$

$\Rightarrow$ It is non differentiable at the points $\{-1, 0, 1\}$.

5.(C) Let a point on $y^3 = x^4$ be (t^3, t^4)

$3y^2 y' = 4x^3 \quad \Rightarrow \quad y' = \dfrac{4x^3}{3y^2} \Rightarrow \quad y' = \dfrac{4}{3}t$

Equation of tangent is $y - t^4 = \dfrac{4t}{3}(x - t^3)$

$\because$ it is a normal to $x^2 + y^2 - 2x = 0$

$\therefore$ it must pass through (1, 0)

$\Rightarrow \quad -\dfrac{3}{4}t^3 = 1 - t^3 \Rightarrow \dfrac{t^3}{4} = 1 \quad \Rightarrow \quad t^3 = 4 \qquad$ Now $m = \dfrac{4t}{3} \quad \Rightarrow \quad \left(\dfrac{3m}{4}\right)^3 = t^3 = 4$

6.(B) Since $\lim\limits_{x \to 1}\left(x^2 - 1\right) = 0$, so limit will exist only, when $\lim\limits_{x \to 1}\left[a\sin(x-1) + b\cos(x-1) + 4\right] = 0$

$\Rightarrow \quad b + 4 = 0 \Rightarrow b = -4$

Now applying L'Hospital rule, we get :

$$\lim_{x \to 1} \frac{a\cos(x-1) - b\sin(x-1)}{2x} = -2 \quad \Rightarrow \quad \frac{\to a}{\to 2} = -2 \quad \Rightarrow \quad a = -4$$

7.(A) $f(x) = x^3 + px^2 + qx + 1$

$f(0) = 1 > 0;\ f(-1) = p - q < 0 \qquad \therefore \qquad x_0 \in (-1, 0)$

$$2\tan^{-1}\left(\frac{1}{\sin x_0}\right) + \tan^{-1}\left(\frac{2\sin x_0}{1 - \sin^2 x_0}\right)$$

$$2\left(\tan^{-1}\left(\frac{1}{\sin x_0}\right) + \tan^{-1}\left(\sin x_0\right)\right)$$

$$2\left(-\frac{\pi}{2}\right) = -\pi \ \left(\because \sin x_0 < 0\right)$$

SECTION-3

8.(A) A and B are independent events.

$$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{12}$$

$$P(A \cap \overline{B}) = P(A) \cdot P(\overline{B}) = \frac{1}{3} \times \left(1 - \frac{1}{4}\right) = \frac{1}{4}$$

$$P(\overline{A} \cap B) = P(\overline{A}) \cdot P(B) = \left(1 - \frac{1}{3}\right) \cdot \frac{1}{4} = \frac{1}{6}$$

(A) $\quad P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{\frac{1}{12}}{\frac{1}{4}} = \dfrac{1}{3} = \lambda_1$

$\therefore \ 12\lambda_1 = 4$ [natural number and composite number]

(B) $\quad P\left(\dfrac{A}{A \cup B}\right) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)}$

$$= \frac{P(A)}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B)} - P(A \cap B)$$

$$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{4} - \frac{1}{12}} = \frac{2}{3} = \lambda_2 \qquad \therefore\ 9\lambda_2 = 6$$

[natural number, composite number and perfect number]

(C) $P(A \cap \bar{B}) \cup (\bar{A} \cap B)) = P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$$= \frac{1}{4} + \frac{1}{6} = \frac{5}{12} = \lambda_3$$

$\therefore\ 12\lambda_3 = 5$ [prime number and natural number]

(D) $P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B)$

$$= \left(1 - \frac{1}{3}\right) + \frac{1}{4} - \frac{1}{6} = \frac{3}{4} = \lambda_4$$

$\therefore\ 12\lambda_4 = 9$ [natural number and composite number]

9.(C) (A) $f(x) = \begin{vmatrix} x^2 & 2x & 1+x^2 \\ x^2+1 & x+1 & 1 \\ x & -1 & x-1 \end{vmatrix}$

Applying $C_1 \to C_1 - C_3$, then

$$f(x) = \begin{vmatrix} -1 & 2x & 1+x^2 \\ x^2 & x+1 & 1 \\ 1 & -1 & x-1 \end{vmatrix}$$

Expanding along R_1, then

$$f(x) = -(x^2 - 1 + 1) - 2x(x^3 - x^2 - 1) + (1 + x^2)(-x^2 - x - 1)$$

$$= -3x^4 + x^3 - 3x^2 + x - 1 \qquad \text{...(i)}$$

According to the question, we get

$$f(x) = a_0 x^4 + a_1 x^3 + a_2 x^2 + a_3 x + a_4 \qquad \text{...(ii)}$$

From equation (i) and (ii), we get

$a_0 = -3,\ a_1 = 1,\ a_2 = -3,\ a_3 = 1,\ a_4 = -1$

(A) $a_0^2 + a_1 = (-3)^2 + 1 = 9 + 1 = 10 = 2 \times 5$

(B) $a_2^2 + a_4 = (-3)^2 - 1 = 9 - 1 = 8 = 2 \times 4$

(C) $a_0^2 + a_2 = (-3)^2 - 3 = 9 - 3 = 6 = 2 \times 3$

(D) $a_4^2 + a_3^2 + a_1^2 = (-1)^2 + (1)^2 + (1)^2 = 1 + 1 + 1 = 3$

10.(B) $f(x)=\sin^{-1}(2x-1)+\cos^{-1}\left(2\sqrt{x-x^2}\right)+\tan^{-1}\left(\frac{1}{1+[x^2]}\right)$

Domain of $f(x)=[0,1]$

$$f(x)=\begin{cases} 0+\frac{\pi}{4}=\frac{\pi}{4} & , \quad x\in\left[0,\frac{1}{2}\right] \\ 2\sin^{-1}(2x-1)+\frac{\pi}{4} & , \quad x\in\left(\frac{1}{2},1\right) \\ \pi+\tan^{-1}\left(\frac{1}{2}\right) & , \quad x=1 \end{cases}$$

Now, verify the options.

11.(D) (A) $\int_1^3 g(x)\,dx+\int_3^1 g^{-1}(x)\,dx=0$

(B) $f(x)\big|_{\max.}=f(\pi)=\frac{5+1}{3-1}=3$

(C) $f'(x)=3x^2+2px+q\Big/_{\!\!\!\!\!\!\!\!\!\!\!\!3}^{1}$

$\frac{q}{3}=3 \quad\Rightarrow\quad q=9$

$\frac{-2p}{3}=4\Rightarrow p=-6 \quad\Rightarrow\quad p+q=3$

(D) $L=\int_0^1 \frac{dx}{(1+x)(2+x)} \quad =\int_0^1\left(\frac{1}{1+x}-\frac{1}{2+x}\right)dx=\left(\ln\left|\frac{1+x}{2+x}\right|\right)_0^1$

$\Rightarrow \quad \ln\left(\frac{2}{3}\right)-\ln\left(\frac{1}{2}\right) \quad\Rightarrow\quad \ln\left(\frac{4}{3}\right)\equiv\ln\left(\frac{a}{b}\right) \quad\therefore\quad |a-b|=1$

SECTION-4

1.(19) Using Lagrange's mean value theorem, for some $c\in(1,\,6)$

$f'(c)=\frac{f(6)-f(1)}{5}=\frac{f(6)+2}{5}\geq 4.2$ or $f(6)+2\geq 21$ or $f(6)\geq 19$

2.(0) $f(2x^2-1)=(x^3+x)f(x)$ ……(1)

Replacing x by $-x$

$f(2x^2-1)=-(x^3+x)f(-x)$ ……(2)

From (1) and (2), we get : $f(-x)=-f(x)$

Hence $f(x)$ is an odd function and as it is continuous,

$\Rightarrow \quad f(0)=0$

Now, $\lim\limits_{x\to0}\frac{f(2x^2-1)}{x}=\lim\limits_{x\to0}(x^2+1)f(x)$

$\lim\limits_{x\to0}f(x)=0$ ($f(x)$ is continuous at $x=0$)

$\Rightarrow \quad \lim_{x\to 0} \frac{f(2x^2-1)}{x} = 0$

Let $\quad x = \sin\frac{\theta}{2}; \qquad \lim_{\theta\to 0} \frac{f\left(2\sin^2\frac{\theta}{2}-1\right)}{\sin\frac{\theta}{2}} = -\lim_{\theta\to 0} \frac{f(\cos\theta)}{\sin\frac{\theta}{2}} = 0 \qquad \ldots\ldots(3)$

$\Rightarrow \quad \lim_{x\to 0} \frac{f(\cos x)}{2\sin\frac{x}{2}\cos\frac{x}{2}} = 0 \ \Rightarrow \quad \lim_{x\to 0} \frac{f(\cos x)}{\sin x} = 0 \qquad$ (Using (3))

3.(0) $\quad e^{-\sqrt{|\ln\{x\}|}} - \{x\}^{1/\sqrt{|\ln\{x\}|}} = 0, \quad \forall\ x \notin Z$

$$[\text{sgn}\ (x)] = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

$\ln\{x\}$ is well defined only when $x \notin Z$

$\Rightarrow \ \{x\} \in (0, 1) \qquad \Rightarrow \ln\{x\} \in (-\infty, 0)$

$\Rightarrow |\ln\{x\}| \in (0, \infty) \qquad \Rightarrow \ e^{-\sqrt{|\ln\{x\}|}} \in (0, 1)$

And $\{x\}^{1/\sqrt{|\ln\{x\}|}} \in (0, 1)$

Because as : $\quad \{x\}: \ 0 \to 1$

$$\frac{1}{\sqrt{|\ln\{x\}|}}: \ 0 \to \infty$$

Hence: $e^{-\sqrt{|\ln\{x\}|}} - \{x\}^{1/\sqrt{|\ln\{x\}|}} \in (-1, 1)$

$[\text{sgn}(x)]$ takes values $\{0, \pm 1\}$

However, $[\text{sgn}(x)] = 0$ iff $x = 0$

Which isn't possible here

Hence, the given equation has no solutions

4.(3) $\quad \lim_{\alpha\to 0}\left(\frac{\ln(1+\tan\alpha^n)}{(\cos(\alpha^m)-1}\right) = -2$

$$\lim_{\alpha\to 0}\left(\frac{\ln(1+\tan\alpha^n)}{\tan(\alpha^n)} \cdot \frac{(\alpha^m/2)^2}{(-2\sin^2(\alpha^m/2))} \cdot \frac{\tan(\alpha^n)}{(\alpha^m/2)^2}\right) = -2$$

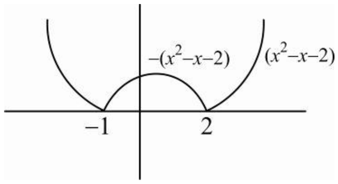
$1 \times \frac{1}{-2}\left[\lim_{\alpha\to 0}\left(\frac{\tan(\alpha^n)}{\alpha^n}\right) \times \frac{\alpha^n}{\alpha^{2m}} .2^2\right] = -2; \qquad \lim_{\alpha\to 0} \alpha^{(n-2m)} = 1$

$n = 2m; \qquad \frac{n}{m} = 2$

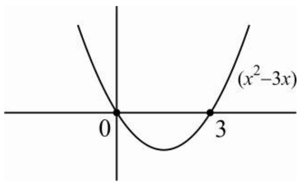
Hence $\frac{n+m}{m} = 2 + 1 = 3$

5.(3) $x^2 - x - 2 = (x-2)(x+1)$

$\therefore$ Graph of $|x^2 - x - 2|$ is as follows :

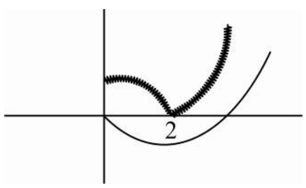


Graph of $x^2 - 3x = x(x-3)$

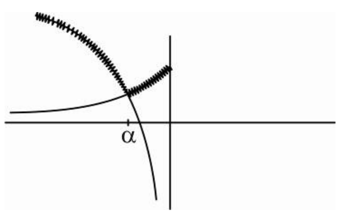


Now $(x^2 - x - 2) - (x^2 - 3x) = 2x - 2 > 0 \ \forall \ x > 2$

$\therefore$ $\max\left(|x^2 - x - 2|, x^2 - 3x\right)$ is as follows (the shaded part)



Now $\max\left(\ell n(-x), e^x\right) \ x < 0$ is as follows (the shaded part)



The function is clearly Non-differentiable at $x = \alpha$ and $x = 2$

At $x = 0$, RHL $= f(0) = 2$ and LHL $= \lim\limits_{x \to 0^-} e^x = 1$

$\therefore$ Function is Non-diff. at $x = 0$ $\therefore$ Total of 3 points.

6.(3) $f(y) \cdot f\left(\frac{x}{y}\right) = f(x)$. Let $x = y = 1 \Rightarrow [f(1)]^2 = f(1) \Rightarrow f(1) = 1 \quad [\because f(1) \neq 0]$

Differentiating w.r.t. x keeping y constant $f(y) \cdot f'\left(\frac{x}{y}\right) \times \frac{1}{y} = f'(x)$. Let $y = x$

$\Rightarrow \quad f(x) \cdot f'(1) \times \frac{1}{x} = f'(x) \quad \Rightarrow \quad \frac{f'(x)}{f(x)} = \frac{3}{x} \Rightarrow \log f(x) = 3 \log x + \log k$

$\Rightarrow \quad f(x) = k x^3 \quad \because f(1) = 1 \Rightarrow f(x) = x^3$

$x^3 = x \quad \Rightarrow \quad$ 3 solutions $-1, 0, 1$